

WESTFIELD PRIMARY SCHOOL

# Maths - Progression in Calculations Policy

April 2023

Review date: April 2024

This Policy was written by J Phillips  
Reviewed by K Beattie



**Progression in calculation:**

This document details the progression in calculation appropriate for each year group, as outlined in the National Curriculum.

Children's fluency in calculation should be given greater emphasis than their ability to use an informal or formal written method. This is particularly important to note as teachers may be tempted to move children on to the next stage before they are fluent.

**Strategies:**

The strategies that children should be aware of are detailed for each year group.

Children should be encouraged to use a range of strategies and to consider the most appropriate strategy for any given calculation. Children's ability to consider a range of strategies should be given greater emphasis than their ability to use a particular informal or formal written method.

**Models and Images:**

At all stages of calculation, children need experience of concrete materials and to understand the connection between this and abstract forms of representation. We follow the Concrete, Pictorial, Abstract method of teaching in Maths. (see appendix for examples for each of the four operations)

**Reasoning and Problem Solving:**

The National Curriculum for Mathematics aims to ensure that all pupils reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing arguments, justification or proof using mathematical language so that they can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Reasoning and Problem solving should be a key element which is interwoven into maths lessons throughout the year groups and stages of calculation. Children should be given a variety of rich problems to increase their fluency and understanding at a particular stage. The language of Maths is a key aspect to children's fluency and understanding. Therefore, the key vocabulary must be explicitly taught and revised regularly. Children must be given opportunities to explain their processes and understanding. (see appendix for key vocabulary lists)

Mathematics is an interconnected subject in which pupils need to move fluently between representations of mathematical ideas. The policy is, by necessity, organised into apparently distinct domains but pupils should be encouraged in all lessons to make rich connections across mathematical ideas.

### **Reasons for using written methods**

- To aid mental calculation by writing down some of the numbers and answers involved.
- To make clear a mental procedure for the pupil.
- To help communicate methods and solutions.
- To provide a record of work to be done.
- To aid calculation when the problem is too difficult to be done mentally.
- To develop and refine a set of rules for calculation.

### **When are children ready for written calculations?**

#### **Addition and subtraction:**

- Do they know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers?
- Do they understand the relationship between addition and subtraction?
- Can they add three single digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

#### **Multiplication and division:**

- Do they know the 2, 3, 4, 5, and 10 times tables?
- Do they know the result of multiplying by 0 and 1?
- Do they understand 0 as a place holder?
- Can they multiply two and three digit numbers by 10 and 100?
- Can they double and halve two digit numbers mentally?
- Can they use multiplication facts to derive mentally other multiplication facts?

The expectation is that the majority of pupils will move through the policy at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage (stage before age). Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. This will ensure fluency however those who are not sufficiently fluent at a particular stage should not be progressed.

**It is essential that children's mental methods in all four operations are secure and they are able to use a variety of strategies as appropriate.**

## Progression in Calculations

### Early Years Expectations

By the end of the year pupils should:

#### Addition and subtraction

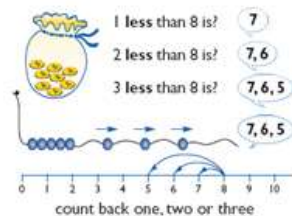
- estimate how many objects they can see and check by counting them
- use the language of more or fewer to compare 2 sets
- count reliably with numbers from 1 to 20, place them in order and say which number is one more or less than a given number
- find the total number of items in two groups by counting all of them
- begin to use the vocabulary involved in adding and subtracting in practical activities and discussion
- record using marks that they can interpret and explain
- use quantities or objects to add and subtract 2 single digit numbers and count on or back to find the answer
- begin to identify own mathematical problems based on own interests and fascinations
- *explore and solve problems in a range of practical and play contexts utilising own methods*

We need 3 milks. We have 4 so I need to take one away

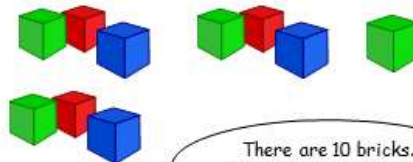


In practical activities and discussion beginning to use the vocabulary involved in adding and subtracting

Finds one more or one less than a given number to 20



Begins to identify own mathematical problems based on own interests and fascinations



There are 10 bricks.  
5 for you and 5 for me

familiar contexts such as number rhymes or stories

trying in a pan ...



10, 9, 8, 7, ...

Continue the count back in ones from any given number

Begin to relate subtraction to 'taking away'



Three teddies **take away** two teddies leaves one teddy

## **Progression in Calculations**

### **Addition and Subtraction**

#### **Year 1 Expectations**

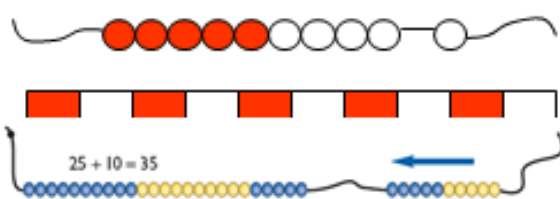
- identify and represent numbers using objects and pictorial representations including the numberline.
- use the language of equal to, more than, less than, most and least
- count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number.
- given a number, identify one more and one less.
- identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least.
- Represent and use number bonds and related subtraction facts within 20, in several forms i.e.  $7 + 9 = 16$ ;  $7 = 16 - 9$ ;  $16 - 7 = 9$ . This establishes addition and subtraction as related operations.
- add and subtract one-digit and two-digit numbers to 20, including zero.
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems.
- pupils should realise the effect of adding or subtracting zero.
- pupils combine and increase numbers, counting forwards and backwards. They discuss and solve problems in familiar practical contexts, including using quantities.

NB- children would only begin recording number sentences using symbols once they have a secure understanding of numbers and related number bonds

#### **Resources to be used include:**

- Numicon
- Bar Model
- Part, part whole representations
- Cubes
- Dice
- Bead strings
- Numberlines
- Counters
- Number cards
- The outside learning environment

## Addition



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

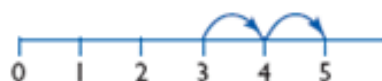
Count in ones and tens

Begin to relate addition to combining two groups of objects



and makes 5

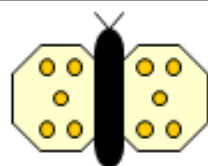
$$3 + 2 = 5$$



Count along a number line to add numbers together

Begin to use the + and = signs to record mental calculations in a number sentence

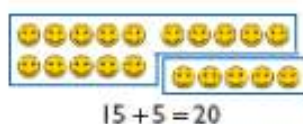
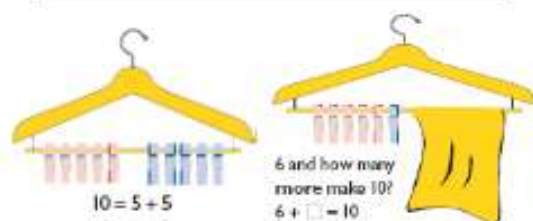
$$6 + 4 = 10$$



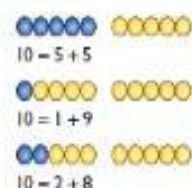
$$5 + 5 = 10$$

Know doubles of numbers 10

Know by heart all pairs of numbers with a total of 10 and 20



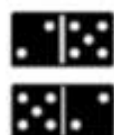
$$5 + ? = 10$$



$$1 + 2 = 3$$



$$2 + 1 = 3$$



$$2 + 5 = 7$$

$$5 + 2 = 7$$

2 count on 5



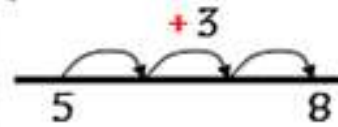
5 count on 2

Know that addition can be done in any order

Put the largest number first and count on- mental & jottings



$$3 + 5$$

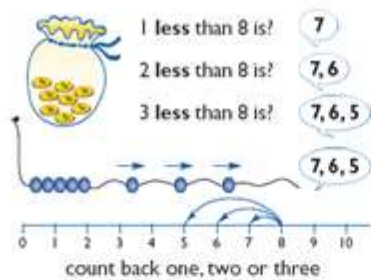


$$8 + 7 = 15$$

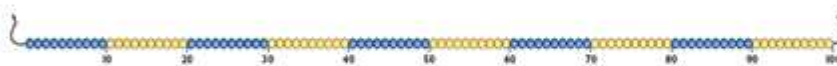


Add two single-digit numbers that bridge 10 (Bridging)

## Subtraction



Find one less than a number



Count back in tens



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



If I take away four shells there are six left

Count backwards along a number line to 'take away'

Begin to use the - and = signs to record mental calculations in a number sentence





Maria had six sweets and she ate four. How many did she have left?



$$6 - 4 = 2$$



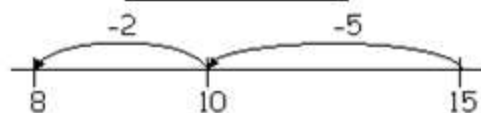
$$\begin{array}{l} 6 + ? = 10 \\ 10 - 6 = ? \end{array} \quad \begin{array}{l} ? + 6 = 10 \\ 10 - 4 = 6 \end{array}$$

		$20 = 12 + 8$	$8 + 12 = 20$
		$20 - 8 = 12$	$20 - 12 = 8$

Know by heart subtraction facts for numbers up to 10 and 20

Subtract single digit numbers often bridging through 10

$$15 - 7 = 8$$





## **Progression in Calculation:**

### **Addition and Subtraction**

#### **Year 2 Expectations**

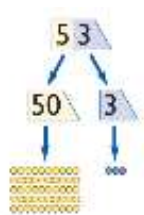
- recognise the place value of each digit in a two-digit number (tens, ones).
- solve problems with addition and subtraction, using concrete objects and pictorial representations, including those involving numbers, quantities and measures.
- Solve problems with addition and subtraction applying my knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones.
  - a two-digit number and tens.
  - two two-digit numbers.
  - adding three one-digit numbers.
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot.
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

#### **Resources to be used include:**

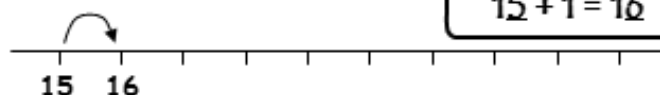
- Numicon
- Bar Model
- Part, part whole representations
- Cubes
- Dice
- Bead strings
- Counters
- Number cards
- The outside learning environment
- Dienes
- Place value cards and counters
- 100 squares
- Numberlines

## Addition

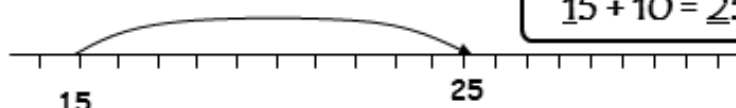
Begin to partition numbers in order to add



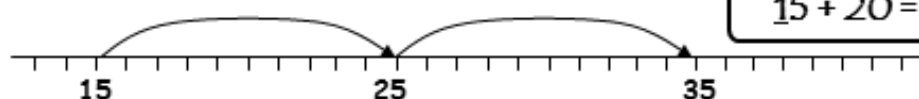
Know which digit changes when adding 1s or 10s to any number



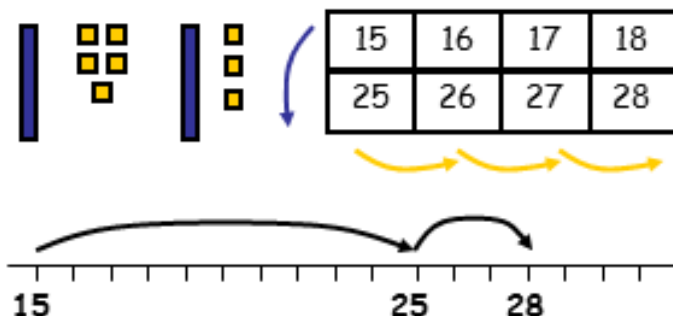
$$15 + 1 = 16$$



$$15 + 10 = 25$$



$$15 + 20 = 35$$



Adding two two-digit numbers (without bridging)

Counting in tens and ones

Partitioning and recombining

N.B. Vertical & horizontal scales

$$15 + 13 = 28$$

Adding two two-digit numbers (bridging through tens boundary)

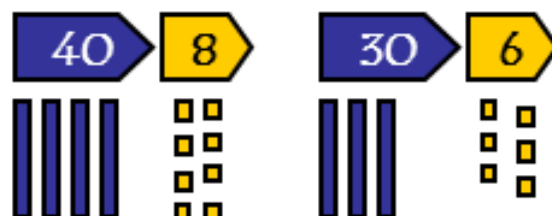
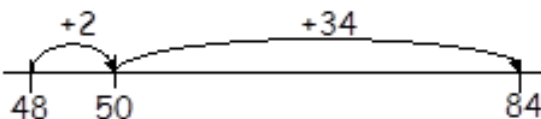
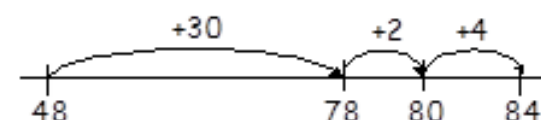
Using a number line

OR

Using place value cards and place value apparatus to partition numbers and recombine

(N.B. Correct use of language-exchanging)

$$48 + 36 = 84$$



$$40 + 30 + 8 + 6$$

$$40 + 30 = 70$$

$$8 + 6 = 14$$

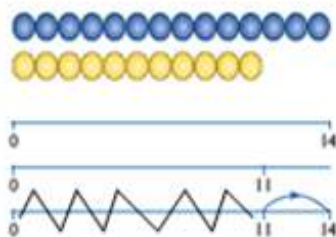
$$70 + 14 = 84$$

## Subtraction

ITP "Difference" to model the concept



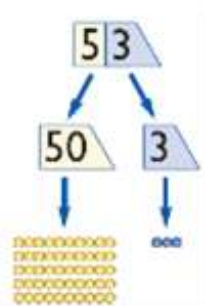
The difference is?



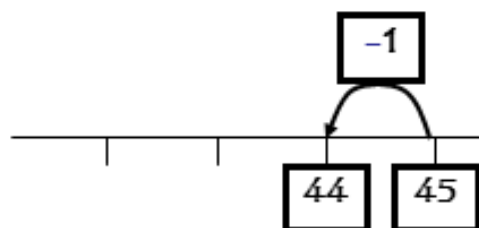
The difference between 11 and 14 is 3.  
 $14 - 11 = 3$   
 $11 + \square = 14$

Begin to find the difference by counting up from the smallest number- shop keeper method

Begin to partition numbers in order to take away



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



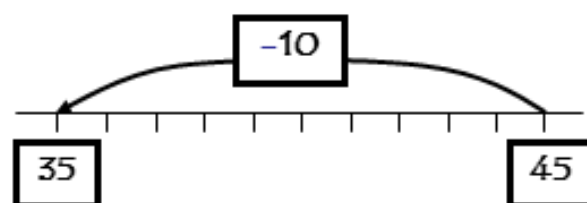
Subtract 1 from a two-digit number mentally. Model by counting on and counting back as appropriate

$$45 - 1$$

Subtract 10 from a two-digit number mentally. Model by counting on and back

$$45 - 10$$

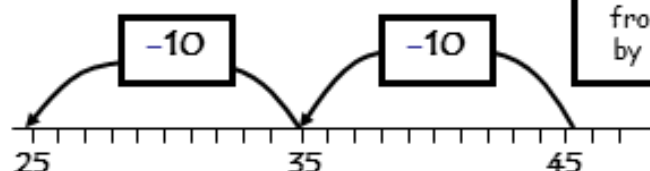
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



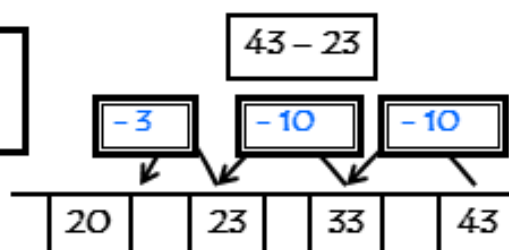
Subtract multiples of 10 from any number. Model by counting on and back

$$45 - 20$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

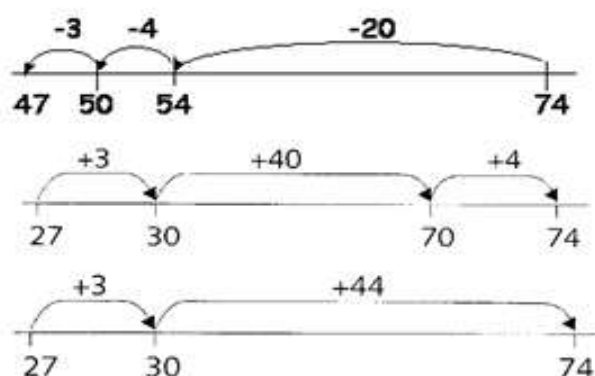


Partition the number to be subtracted (no exchanging)



$$43 - 20 = 23$$

$$23 - 3 = 20$$



Decide whether to count on or count back

$$74 - 27 = 47$$

Now what's the answer?

17

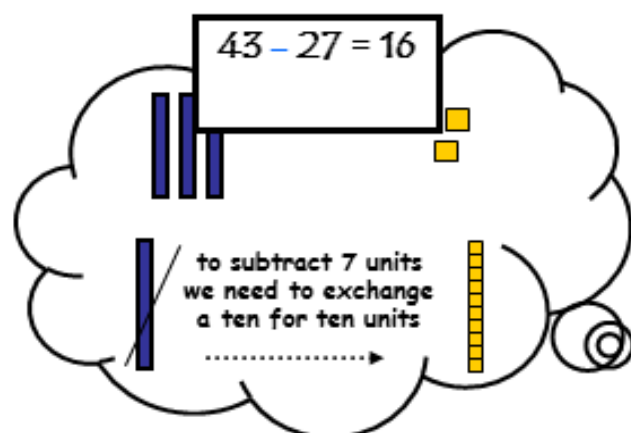
Partitioning number to be subtracted - with exchanging (links to counting back on number line)

$$43 - 27 = 16$$



$$\begin{array}{r} 43 - 20 = 23 \\ 23 - 7 = 16 \end{array}$$

?



Expanded method

It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.

T	U
- 2	7

$$\begin{array}{r} 30 \cancel{4}0 + 10 + 3 \\ - 20 + 7 \\ \hline 10 + 6 \end{array}$$

**Standard written method**

The previous stages reinforce what happens to numbers when they are subtracted using more formal written methods. It is important that the children have a good understanding of place value and partitioning.

$$\begin{array}{r} 3 \cancel{4} 13 \\ - 27 \\ \hline 16 \end{array}$$

## **Progression in Calculation:**

### **Addition and Subtraction**

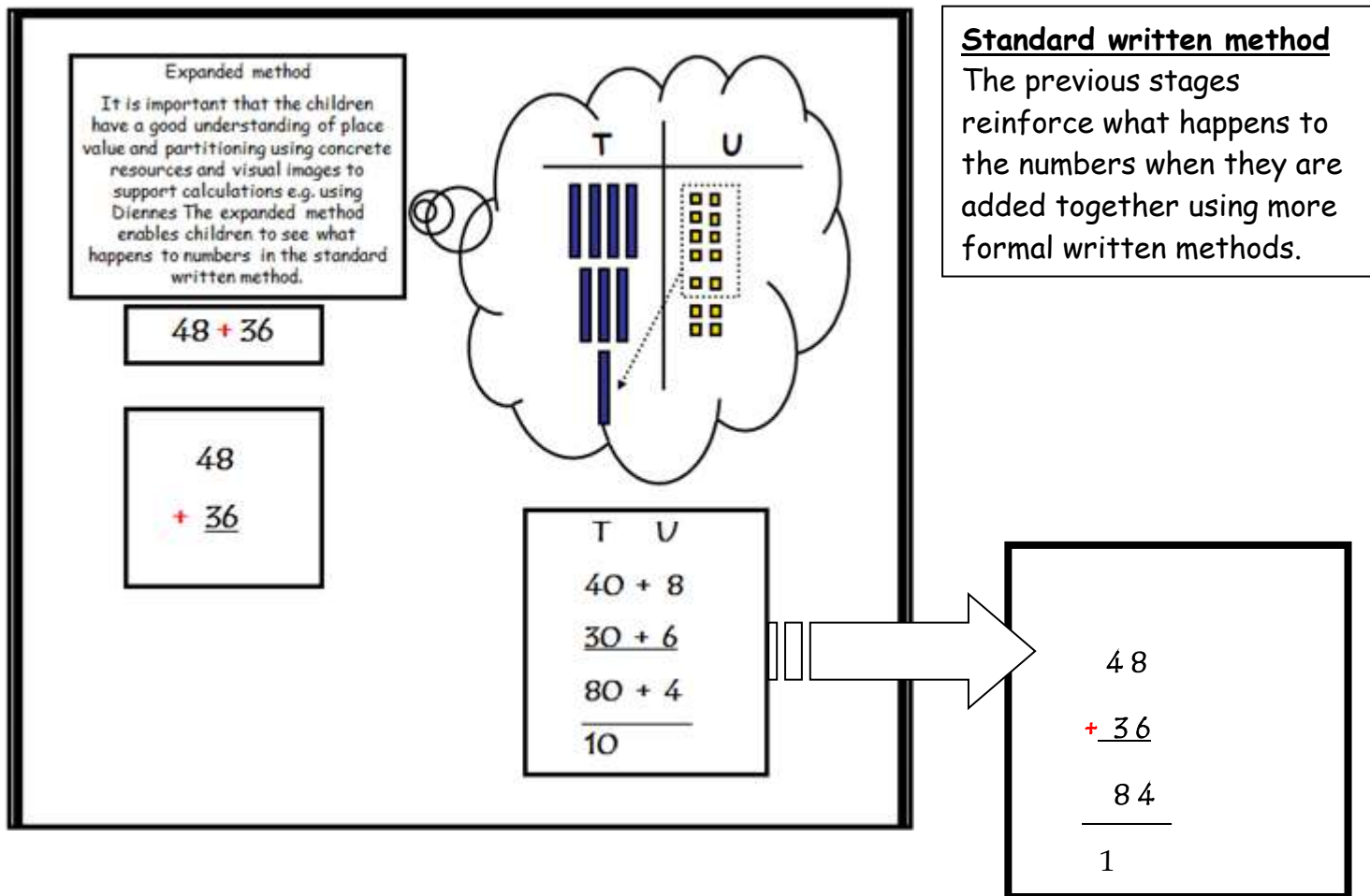
#### **Year 3 Expectations**

- find 10 or 100 more or less than any given number
- recognise the place value of each digit in a 3-digit number (100s, 10s, 1s)
- add and subtract numbers mentally, including:
  - \*a three-digit number and 1s.
  - \*a three-digit number and 10s.
  - \*a three-digit number and 100s.
- pupils use their understanding of place value and partitioning, to use the columnar method for addition and subtraction for 2 digit numbers, increasing to 3 digits by the end of the year (see NC Appendix 1)
- add and subtract fractions, with the same denominator, within a whole
- estimate the answer to a calculation and use inverse operations to check answers.
- solve varied problems, including missing number problems, using number facts, place value and more complex addition and subtraction.
- calculate the answers to mental calculations, where answers can exceed 100

#### **Resources to be used include:**

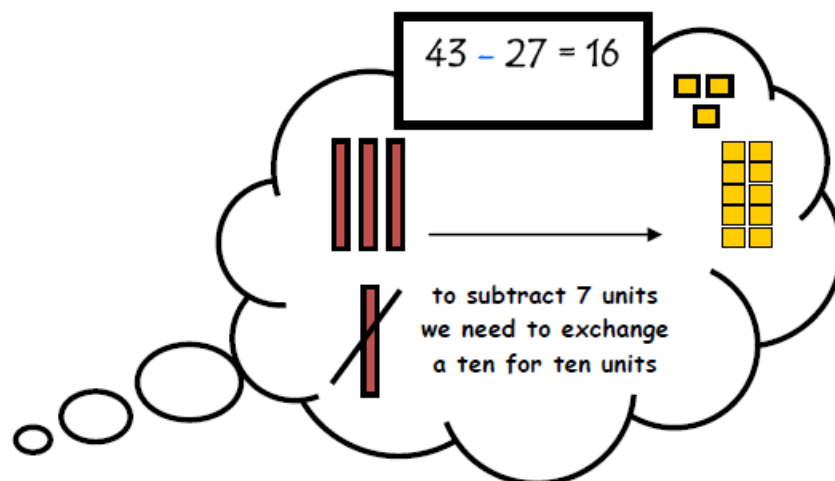
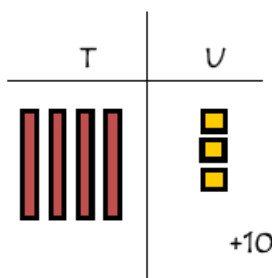
- Bar Model
- Part, part whole representations
- Cubes
- Dice - including place value dice to 3 digits
- Bead strings
- Counters
- Number cards
- The outside learning environment
- Dienes
- Place value cards and counters
- 100 squares
- Numberlines

## Agreed Written Strategies:



## Subtraction:

e.g.  $43 - 27 =$



<del>30</del>	+10	
$40$	+	$3$
$- 20$	+	$7$
<u><math>10</math></u>		<u><math>6</math></u>
	$=$	$16$

- refine expanded methods of column addition and subtraction. Making sure that the link between the expanded method and the compact method are explicit. It is still very important that concrete materials are used for visual learners.

**Standard written method**

The previous stages reinforce what happens to numbers when they are subtracted using more formal written methods. It is important that the children have a good understanding of place value and partitioning.

$$\begin{array}{r} \overset{3}{\cancel{4}} \overset{1}{3} \\ - 27 \\ \hline 16 \end{array}$$

## **Addition and Subtraction**

### **Year 4 Expectations**

#### **Progression in Calculation:**

- recognise the place value of each digit in a four-digit number (1,000s, 100s, 10s, and 1s).
- find 10,100, 1000 more or less than a given number.
- identify, represent and estimate numbers using different representations
- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate. Ensure that children are exposed to calculations involving numbers with varying numbers of digits (e.g.  $453 + 72$ ).
- estimate and use inverse operations to check answers to a calculation.
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.
- Solve number and practical problems with increasingly large positive numbers
- add and subtract fractions with the same denominator.
- add and subtract using decimals within the context of money.

#### **Resources to be used include:**

- Numberlines
- Bar Model
- Part, part whole representations
- Cubes
- Dice - including place value dice to 3 digits
- Bead strings
- Counters
- Number cards
- The outside learning environment
- Dienes
- Place value whiteboards, cards and counters
- 100 squares

**Children need to make appropriate decisions when deciding whether to calculate mentally or to use a written calculation. For example:  $2002 - 1999$  would be most efficiently solved using a mental method.**



### Agreed Written Strategies:

$$\begin{array}{r} 4563 \\ + 1728 \\ \hline 6291 \\ \hline \end{array}$$

1      1

(Progression from Year 3 - refining the process and ensuring children are comfortable to apply when using bigger numbers.)

#### Standard written method

The previous stages reinforce what happens to numbers when they are subtracted using more formal written methods. It is important that the children have a good understanding of place value and partitioning.

$$\begin{array}{r} \overset{3}{\cancel{4}} \overset{1}{3} \\ - 27 \\ \hline 16 \\ \hline \end{array}$$

## **Addition and Subtraction**

### **Year 5**

#### **Progression in Calculation:**

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction). Ensure that children are exposed to calculations involving numbers with varying numbers of digits (e.g.  $1453 + 72$ ).
- add and subtract numbers mentally with increasingly large numbers.
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- add and subtract fractions with the same denominator and with denominators that are multiples of the same number (e.g.  $1/3 + 2/9$ ).
- practise mental calculations with increasingly large numbers to aid fluency (for ex-ample,  $12,462 - 2,300 = 10,162$ ).
- practise adding and subtracting decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places and complements of 1 (e.g.  $0.83 + 0.17 = 1$ ).
- 

#### **Resources to be used include:**

- Numberlines
- Bar Model
- Dice - including place value dice to 4 digits
- Counters
- Number cards
- The outside learning environment
- Dienes
- Place value whiteboards, cards and counters

**Children need to become more confident to make appropriate decisions when deciding whether to calculate mentally or to use a written calculation. For example:  $2002 - 1999$  would be most efficiently solved using a mental method.**

**Agreed Written Strategies:**

Subtraction Example:

$$\begin{array}{r} \overset{3}{\cancel{4}} \overset{1}{3} \\ - 27 \\ \hline 16 \end{array}$$



$$\begin{array}{r} \overset{3}{\cancel{4}} \overset{1}{3} . 5 \\ - 27 . 3 \\ \hline 16 . 2 \end{array}$$

(Progression from Year 4 - refining the processes of addition and subtraction, and ensuring children are comfortable to apply using both bigger and decimal numbers.)

## **Progression in Calculation:**

### **Year 6**

#### **Addition and Subtraction**

- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operations to carry out calculations involving the 4 operations (BODMAS)
- pupils explore the order of operations using brackets, for example,  $2 + 1 \times 3 = 5$  and  $(2 + 1) \times 3 = 9$ .
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- solve multi-step problems involving addition and subtraction.
- use negative numbers in context and calculate intervals across zero
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.
- generate and describe linear number sequences
- express missing number problems algebraically
- use simple formulae

#### **Resources to be used include:**

- Numberlines
- Bar Model
- Dice - including place value dice to 5 digits
- Counters
- Number cards
- The outside learning environment
- Dienes
- Place value whiteboards, cards and counters

**Children need to become more confident to make appropriate decisions when deciding whether to calculate mentally or to use a written calculation. For example:  $2002 - 1999$  would be most efficiently solved using a mental method.**

#### **Agreed Written Strategies:**

-refine compact methods of column addition and subtraction (including using decimals).  
-children to move onto adding and subtracting fractions by finding common denominators, teachers to extend the children to adding and subtracting fractions where neither are multiples of each other. Children should always be encouraged to simplify their answers when working with fractions. (see year 5 for example)

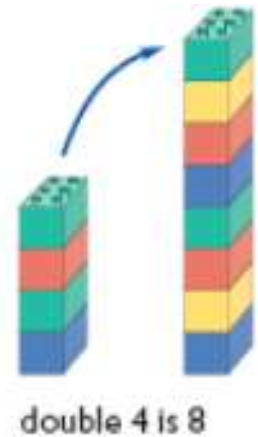
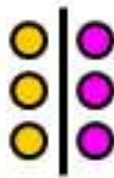
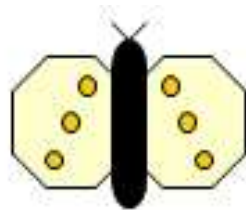
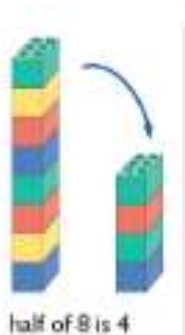
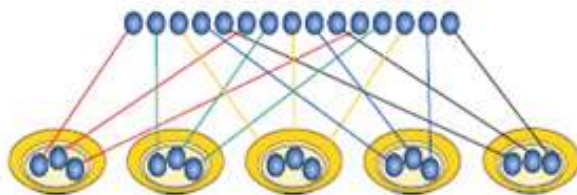
## Progression in Calculation:

### Early Years

By the end of the year children should:

### Multiplication and Division

- record using marks that they can interpret and explain
- solve problems, including doubling, halving and sharing
- begin to identify own mathematical problems based on own interests and fascinations
- *explore and solve problems in a range of practical and play contexts utilising own methods*
- *make two equal groups of objects and check they are equal by counting*



## **Multiplication and Division**

### **Year 1**

#### **Progression in Calculation:**

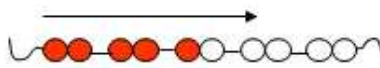
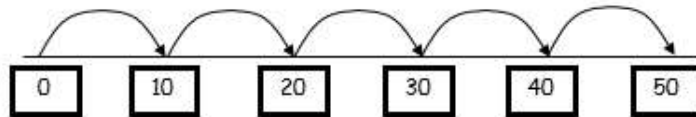
- count, read and write numbers to 100 in numerals.
- count in multiples of twos, fives and tens.
- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.\*
- Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.
- recognise, find and name a half as one of two equal parts of an object, shape or quantity.
- recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.

**\*NB – this area of calculations will not be recorded using symbols and number sentences until year 2, unless the child is working beyond year group expectations in Maths.**

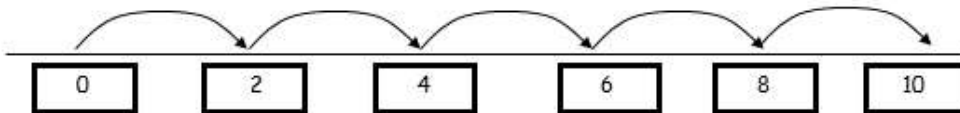
#### **Resources to be used include:**

- Numicon
- Bar Model
- Part, part whole representations
- Cubes
- Dice
- Bead strings
- Numberlines
- Counters
- Magnetic/practical fraction blocks
- Number cards
- The outside learning environment

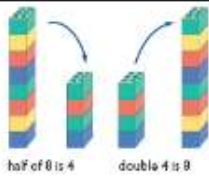
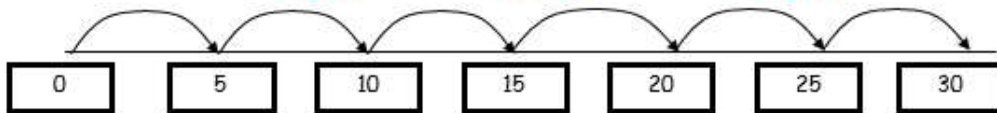
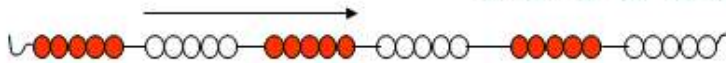
Count in tens from zero



Count in twos from zero



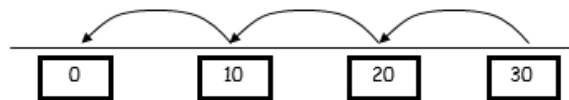
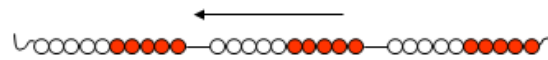
Count in fives from zero



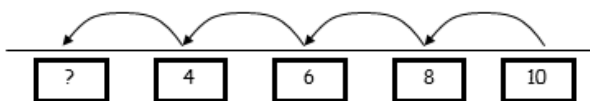
half of 8 is 4  
double 4 is 8

Know doubles and corresponding halves

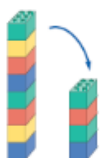
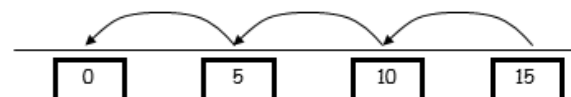
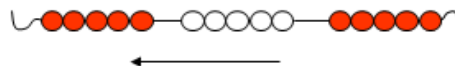
Count back in tens



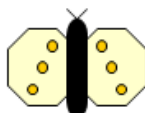
Count back in twos



Count back in fives



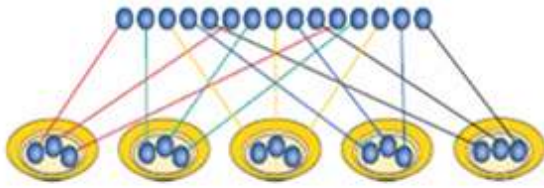
half of 8 is 4  
 $8 \div 2 = 4$



Half of 6 is 3  
 $\frac{1}{2}$  of 6 = 3

Know halves

$15 \div 5 = 3$   
15 shared between 5

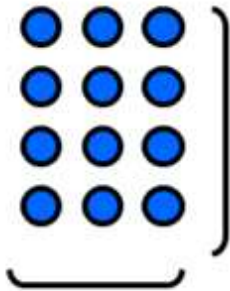
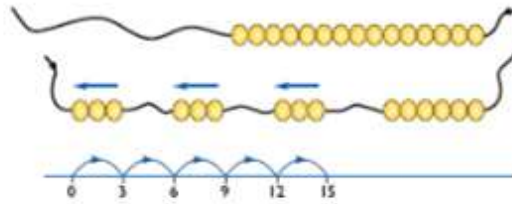


Understand division  
as sharing

Understand division  
as grouping

How many 3s  
in 15?

3 6 9 12 15  
 $15 \div 3 = 5$



12 divided into groups  
of 3 gives 4 groups  
 $12 \div 3 = 4$

12 divided into groups  
of 4 gives 3 groups  
 $12 \div 4 = 3$

Reinforce division as  
grouping through the  
use of arrays



## **Multiplication and Division**

### **Year 2**

#### **Progression in Calculation:**

- count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward.
- Pupils are introduced to the multiplication tables. They recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers. They practise and become fluent in the 2, 5 and 10 multiplication tables.
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals (=) signs.
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
- recognise, find, name and write fractions of a length, shape, set of objects or quantity.
- write simple fractions and recognise equivalents of a half.

#### **Resources to be used include:**

- Numicon
- Bar Model
- Part, part whole representations
- Cubes
- Dice
- Bead strings
- Numberlines
- Counters
- Number cards
- The outside learning environment
- Array cards
- Magnetic/practical fraction blocks

Teach and then pupils know multiplication tables to 10 x 10. x10, x2 and x5 first.

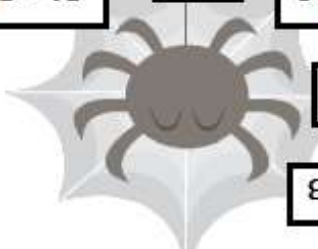
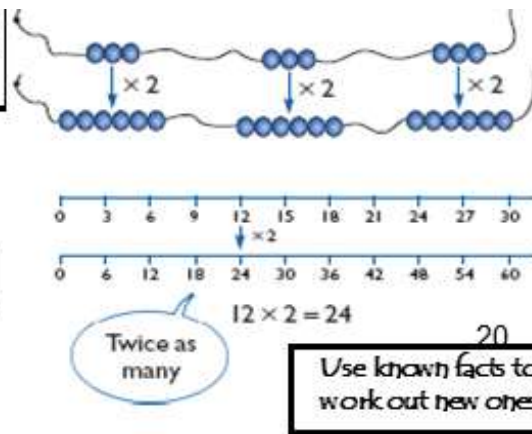
**x 5**

$2 \times 5 = 10$


$6 \times 5 = 30$

$3 \times 5 = 15$

$8 \times 5 = 40$

Understand multiplication as repeated addition



$$2 + 2 + 2 + 2$$

$$2 + 2 + 2 + 2 = 8$$

$$4 \times 2 = 8$$

2 multiplied by 4

4 lots of 2



$$2 \times 4$$



$$4 \times 2 = 8$$

$$2 \times 4 = 8$$



$$4 \times 2$$

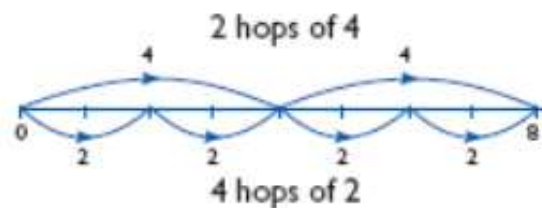


$$2 \times 4 = 8$$

$$4 \times 2 = 8$$

Understand multiplication as an array

Understand how to represent arrays on a number line



## **Multiplication and Division**

### **Year 3**

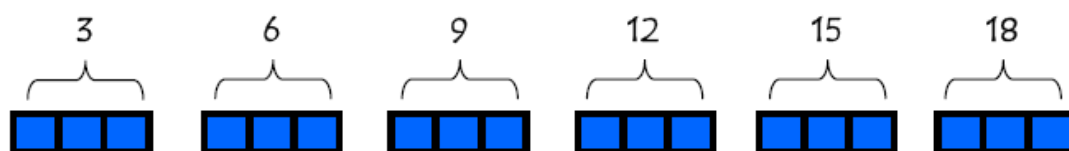
#### **Progression in Calculation:**

- pupils now count in and use, multiples of 2, 3, 4, 5, 8, 10, 50 and 100 (children to use known facts to find multiples of 50 and 100).
- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
- Count up and down in tenths. Recognise that tenths arise from dividing an object into 10 equal parts, from dividing 1-digit numbers or quantities by 10
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods (grid method for multiplication).
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which  $n$  objects are connected to  $m$  objects (for example, if you have 3 hats and 4 coats, how many different outfits can you make?)
- pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables.
- pupils understand the relationship between unit fractions as operators (fractions of) and division by integers (i.e.  $\frac{1}{8}$  of 10 is 10 divided by 8)
- recognise and show, using diagrams, equivalent fractions
- pupils develop efficient mental methods, for example, using commutativity and associativity (for example,  $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$ ) and multiplication and division facts (for example, using  $3 \times 2 = 6$ ,  $6 \div 3 = 2$  and  $2 = 6 \div 3$ ) to derive related facts ( $30 \times 2 = 60$ ,  $60 \div 3 = 20$  and  $20 = 60 \div 3$ ).

#### **Resources to be used include:**

- Bar Model
- Part, part whole representations
- Cubes/counters
- Dice
- Bead strings
- Numberlines
- Magnetic/practical fraction blocks
- Number cards
- The outside learning environment
- Dienes

**Children need to make appropriate decisions when deciding whether to calculate mentally or to use a written calculation. For example:  $12 \times 2$  would be more efficient as a mental calculation.**



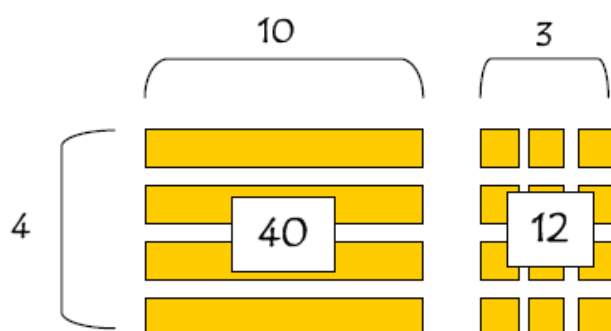
$$18 \div 3 = 6$$



-use practical strategies to lead into more formal written methods. E.g.  $4 \times 13 =$

Use place value apparatus to support the multiplication of  $U \times TU$

$$4 \times 13 =$$



$$40 + 12 = 52$$

-refine the use of the formal written method to multiply two and three digit numbers by one digit. E.g.  $23 \times 4 =$

Ensure the children are aware of the link between the grid method and the formal written method.

$\times$	20	3
4	80	12

$$80 + 12 = 92$$

$$\begin{array}{r} 23 \\ \times 4 \\ \hline 92 \\ \hline 1 \end{array}$$

-progress to multiplying  $TU \times TU$  when ready.

$$100 \div 7 = 70 = 14 \text{ rem } 2$$

$$1 \times 7 = 7$$

$$10 \times 7 = 70$$

$$2 \times 7 = 14$$

$$4 \times 7 = 28$$



### Short division

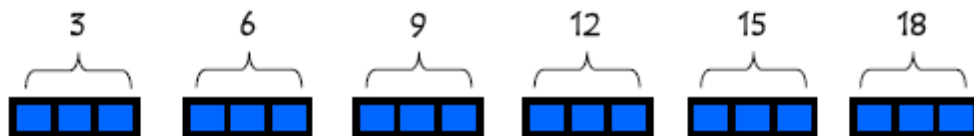
$98 \div 7$  becomes

$$\begin{array}{r} 14 \\ 7 \overline{) 98} \end{array}$$

Answer: 14

18 divided into groups of 3  
 $18 \div 3 = 6$

-represent 'groups' for division on a number line using apparatus alongside the line.



$$18 \div 3 = 6$$



## **Multiplication and Division**

### **Year 4**

#### **Progression in Calculation:**

- count in multiples of 6, 7, 9, 25 and 1,000.
- recall multiplication and division facts for multiplication tables up to  $12 \times 12$ .
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers.
- recognise and use factor pairs and commutativity in mental calculations.
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout.
- solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as  $n$  objects are connected to  $m$  objects. (for example, number of choices of a meal on a menu)
- pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency.
- pupils practise mental methods and extend this to 3-digit numbers to derive facts, (for example  $600 \div 3 = 200$  can be derived from  $2 \times 3 = 6$ ), they combine their knowledge of number facts and rules of arithmetic to solve calculations  $2 \times 6 \times 5 = 10 \times 6 = 60$
- pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers.
- pupils solve two-step problems in contexts
- find the effect of dividing a 1 or 2 digit number by 10 and 100, identifying the value of the digit as 1s, 10<sup>th</sup>s and 100<sup>th</sup>s
- solve problems which involve increasingly harder fractions to calculate quantities. Use fractions to divide quantities, including non-unit fractions where the answer is a whole number.

#### **Resources to be used include:**

- Bar Model
- Magnetic/practical fraction blocks
- Dice
- Bead strings
- Numberlines
- Counters
- Number cards
- The outside learning environment

**Children need to make appropriate decisions when deciding whether to calculate mentally or to use a written calculation. For example:  $12 \times 2$  would be more efficient as a mental calculation.**

## Agreed Written Strategies

### Short multiplication

$24 \times 6$  becomes

$$\begin{array}{r} 24 \\ \times 6 \\ \hline 144 \\ 2 \end{array}$$

Answer: 144

$342 \times 7$  becomes

$$\begin{array}{r} 342 \\ \times 7 \\ \hline 2394 \\ 21 \end{array}$$

Answer: 2394

### Short division

$98 \div 7$  becomes

$$\begin{array}{r} 14 \\ 7 \overline{) 98} \\ \underline{7} \phantom{0} \\ 28 \\ \underline{28} \\ 0 \end{array}$$

Answer: 14

Continuing to secure the formal methods learnt in year 3, dividing 2 and 3 digit numbers by 1 digit, with exact answers.

## **Multiplication and Division**

### **Year 5**

#### **Progression in Calculation:**

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.
- know and use the vocabulary of prime numbers, prime factors and composite (non- prime) numbers. Recall prime numbers up to 19.
- multiply numbers up to 4 digits by a one or two-digit number using a formal written method, including long multiplication for two-digit numbers.
- multiply and divide numbers mentally drawing upon known facts.
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers.
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes. Understand the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.
- Identify, write and name equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as decimals or by rounding (for example,  $98 \div 4 = 24 \text{ r}2 = 24.5 = 25$ ).
- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams, recognise and use 1000ths
- Solve problems which requiring knowing % and decimal equivalents of half, quarters, fifths, two-fifths and four-fifths, and those fractions with a denominator of a multiple of 10, 25 and 50.

#### **Resources to be used include:**

- Bar Model
- Magnetic/practical fraction blocks
- Dice
- Numberlines
- Counters
- Number cards
- The outside learning environment

**Children need to make appropriate decisions when deciding whether to calculate mentally or to use a written calculation. For example:  $12 \times 2$  would be more efficient as a mental calculation.**



## Agreed Written Multiplication Strategies

-use formal written method to multiply TU x TU progressing to HTU x TU. Ensure that the children see the link between the grid method and the formal written method.

$$56 \times 27 =$$

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 392 \quad (56 \times 7) \\ 1120 \quad (56 \times 20) \\ \hline 1512 \end{array}$$

Ensure children are aware of and understand the role of zero as a place holder.

2741 x 6 becomes

$$\begin{array}{r} 2741 \\ \times 6 \\ \hline 16446 \\ 42 \end{array}$$

Answer: 16 446

Children consolidate their use of formal methods of short multiplication, extending to multiplying 4 digits by a 1 digit number.

### Agreed Written Division Methods:

-use the formal written method (bus stop method/ short division) for division. Children should be dividing four digit numbers by single digits.

-children to be giving answers with remainders, as decimals and by using rounding.

$$5715 \div 6 =$$

Answer with a remainder:

$$\begin{array}{r} 0952 \text{ r } 3 \\ 6 \overline{) 5715} \\ \underline{5} \phantom{7} \phantom{1} \phantom{5} \\ 7 \phantom{1} \phantom{5} \\ \underline{6} \phantom{1} \phantom{5} \\ 1 \phantom{5} \\ \underline{0} \phantom{5} \\ 5 \end{array}$$

Answer as a decimal:

$$\begin{array}{r} 0952.5 \\ 6 \overline{) 5715.0} \\ \underline{5} \phantom{7} \phantom{1} \phantom{5} \phantom{.} \phantom{0} \\ 7 \phantom{1} \phantom{5} \phantom{.} \phantom{0} \\ \underline{6} \phantom{1} \phantom{5} \phantom{.} \phantom{0} \\ 1 \phantom{5} \phantom{.} \phantom{0} \\ \underline{0} \phantom{5} \phantom{.} \phantom{0} \\ 5 \phantom{.} \phantom{0} \\ \underline{0} \phantom{0} \\ 0 \end{array}$$

Answer rounded:

953

Children should be made aware that it will depend on the content of the problem as to whether they answer with a decimal, remainder or rounding.

## Multiplication and Division

### Year 6

#### Progression in Calculation:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
- Multiply 1-digit numbers with numbers up to two decimal places
- multiply and divide numbers by 10, 100 and 1000 giving answers up to 3 decimal places
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division (or short division when appropriate, i.e. 2525 divided by 25)
- use written division methods in cases where the answer has up to two decimal places
- interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.
- use common factors to simplify fractions. Use common multiples to express fractions in the same denomination.
- perform mental calculations, including with mixed operations and large numbers.
- identify common factors, common multiples and prime numbers.
- solve problems involving multiplication and division.
- pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.
- pupils explore the order of operations using brackets; for example,  $2 + 1 \times 3 = 5$  and  $(2 + 1) \times 3 = 9$ .
- common factors can be related to finding equivalent fractions.
- associate a fraction with division and calculate decimal equivalents
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example,  $4 \frac{1}{2} \times 2 \frac{1}{2} = 8 \frac{1}{2}$ ].
- divide proper fractions by whole numbers


$\frac{1}{2}$  divided by 3 =  $\frac{1}{6}$

- Associate a fraction with division and calculate decimal equivalents
- multiply one- and two-digit numbers with up to two decimal places by whole numbers, in practical contexts such as money.
- use estimation to check answers to calculations
- solve problems involving the relative sizes of 2 quantities where missing values can be found by using integer multiplication and division facts
- solve problems involving the calculation of percentages [for example, of measures and such as 15% of 360] and the use of percentages for comparison

- solve problems involving similar shapes where the scale factor is known or can be found
- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples
- use simple formulae
- generate and describe linear number sequences
- express missing number problems algebraically
- find pairs of numbers that satisfy an equation with two unknowns
- enumerate possibilities of combinations of two variables

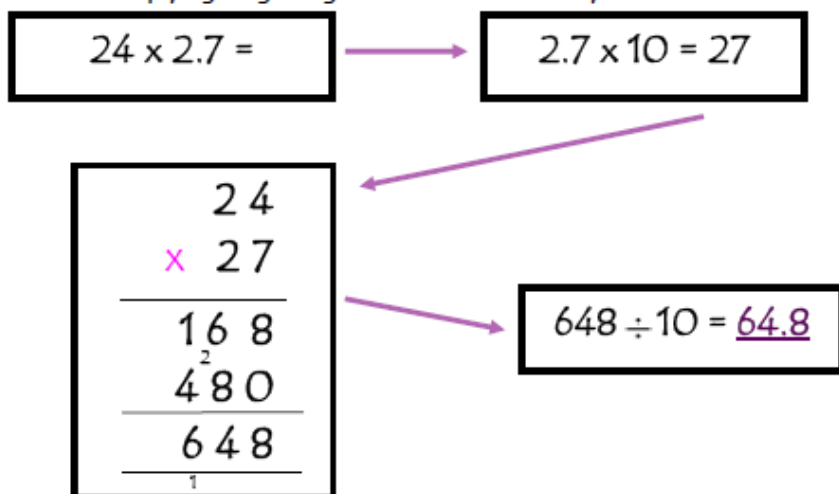
Children need to make appropriate decisions when deciding whether to calculate mentally or to use a written calculation. For example:  $12 \times 2$  would be more efficient as a mental calculation.

**Resources to be used include:**

- Bar Model
- Magnetic/practical fraction blocks
- Dice
- Numberlines
- Counters
- Number cards
- The outside learning environment

#### Agreed Written Multiplication Methods:

- refine formal written method of multiplication, progressing to larger numbers.
- progress to multiplying single digit decimal numbers by whole numbers.



### Agreed Written Division Methods:

- refine formal written method for division, progressing to dividing by two digit number
- children to decide, dependent on the calculation whether, the long or short written method would be most appropriate.

$$432 \div 15 =$$

Short Division:

$$\begin{array}{r} 028 \text{ r } 12 \\ 15 \overline{) 432} \end{array}$$

So What Do I Know...

$$\begin{array}{l} 1 \times 15 = 15 \\ 2 \times 15 = 30 \\ 4 \times 15 = 60 \\ 5 \times 15 = 75 \\ 10 \times 15 = 150 \end{array}$$

Long Division

$$\begin{array}{r} 028 \text{ r } 12 \\ 15 \overline{) 432} \\ \underline{300} \quad (20 \times 15) \\ 132 \\ \underline{120} \quad (8 \times 15) \\ 12 \end{array}$$

# Appendix 1

## Progression in Teaching Addition

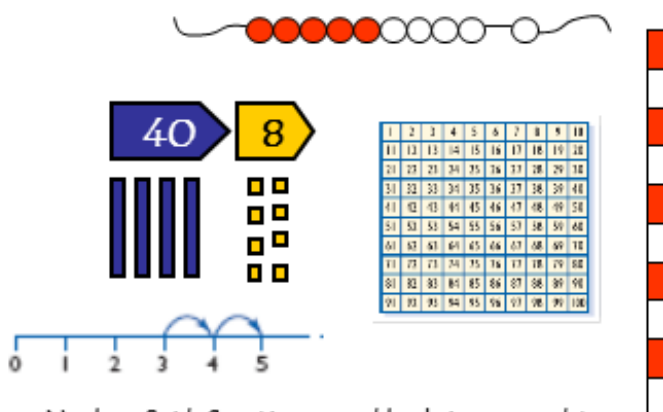
### Mental Skills

Recognise the size and position of numbers  
Count on in ones and tens  
Know number bonds to 10 and 20  
Add multiples of 10 to any number  
Partition and recombine numbers  
Bridge through 10



### Models and Images

Counting apparatus  
Place value apparatus  
Place value cards  
Number tracks  
Numbered number lines  
Marked but unnumbered number lines  
Empty number lines  
Hundred square  
Counting stick  
Bead string  
Models and Images charts  
ITPs - Number Facts, Ordering Numbers, Number Grid, Counting on and back in ones and tens



### Key Vocabulary

add  
addition  
plus  
and  
count on  
more  
sum  
total  
altogether  
increase

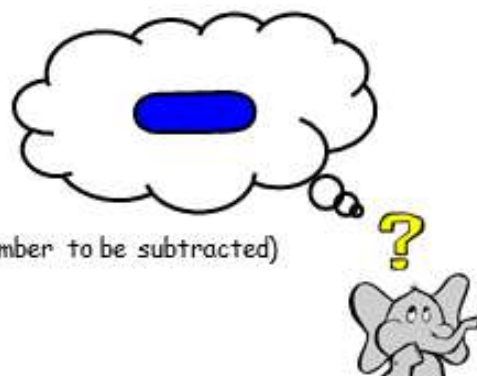
count on +  
add and  
addition plus  
more sum total  
altogether increase

## Appendix 2

### Progression in Teaching Subtraction

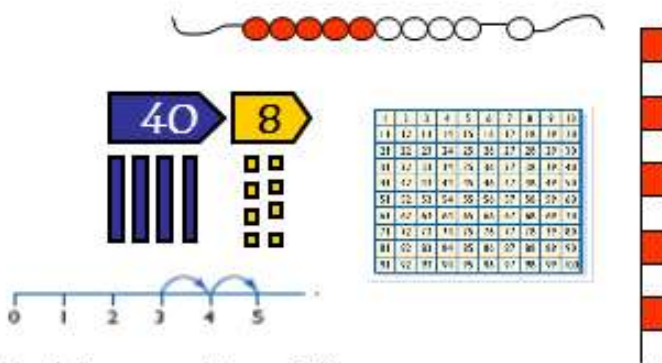
#### Mental Skills

Recognise the size and position of numbers  
Count back in ones and tens  
Know number facts for all numbers to 20  
Subtract multiples of 10 from any number  
Partition and recombine numbers (only partition the number to be subtracted)  
Bridge through 10



#### Models and Images

Counting apparatus  
Place value apparatus  
Place value cards  
Number tracks  
Numbered number lines  
Marked but unnumbered lines  
Hundred square  
Empty number lines  
Counting stick  
Bead strings  
Models and Images Charts  
ITPs - Number Facts, Counting on and back in ones and tens, Difference



#### Key Vocabulary

subtract  
take away  
minus  
count back  
less  
fewer  
difference between  
Decrease  
Exchanging

count back   take away  
fewer   subtract  
minus   less  
difference between

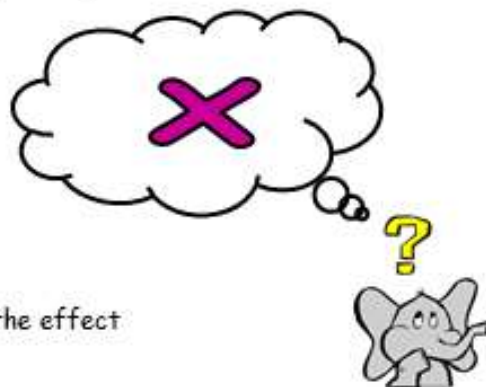


## Appendix 3

### Progression in Teaching Multiplication

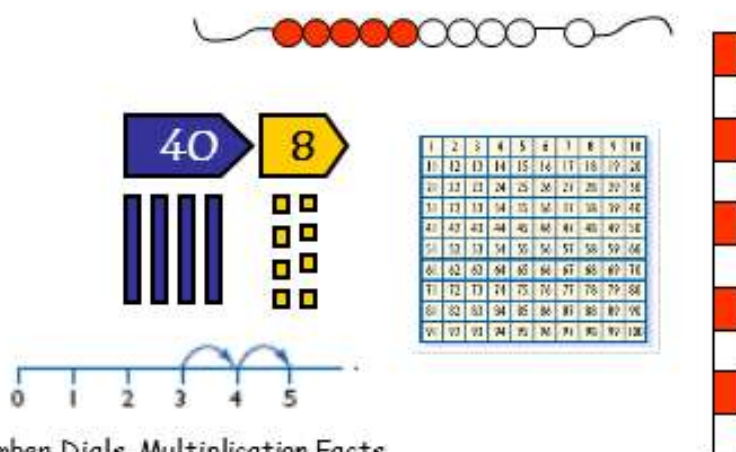
#### Mental Skills

Recognise the size and position of numbers  
Count on in different steps 2s, 5s, 10s  
Double numbers up to 10  
Recognise multiplication as repeated addition  
Quick recall of multiplication facts  
Use known facts to derive associated facts  
Multiplying by 10, 100, 1000 and understanding the effect  
Multiplying by multiples of 10



#### Models and Images

Counting apparatus  
Place value apparatus  
Arrays  
100 squares  
Number tracks  
Numbered number lines  
Marked but unnumbered lines  
Empty number lines  
Multiplication squares  
Counting stick  
Bead strings  
Models and Images charts  
ITPs - Multiplication grid, Number Dials, Multiplication Facts



#### Vocabulary

lots of  
groups of  
times  
multiply  
multiplication  
multiple  
product  
once, twice, three times  
array, row, column  
double  
repeated addition

**multiplication**      **product**  
once, twice, three times  
**double**      **groups of**  
repeated addition      **lots of**  
array, row, column      **multiply**  
**times**      **multiple**

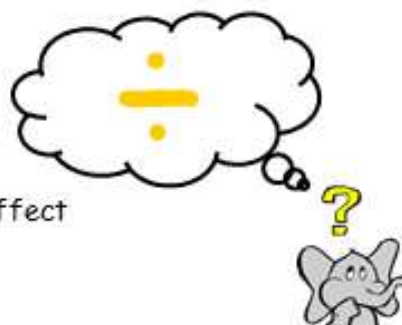


## Appendix 4

### Progression in Teaching Division

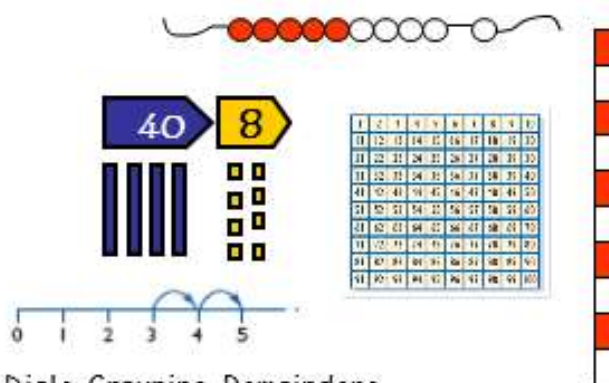
#### Mental Skills

Recognise the size and position of numbers  
 Count back in different steps 2s, 5s, 10s  
 Halve numbers to 20  
 Recognise division as repeated subtraction  
 Quick recall of division facts  
 Use known facts to derive associated facts  
 Divide by 10, 100, 1000 and understanding the effect  
 Divide by multiples of 10



#### Models and Images

Counting apparatus  
 Arrays  
 100 squares  
 Number tracks  
 Numbered number lines  
 Marked but unnumbered lines  
 Empty number lines  
 Multiplication squares  
 Models and Images charts  
 ITPs - Multiplication grid, Number Dials, Grouping, Remainders



#### Vocabulary

lots of  
 groups of  
 share  
 group  
 halve  
 half  
 divide  
 division  
 divided by  
 remainder  
 factor

Quotient- whole number, part of  
 answer e.g. 4 in 4 r1.

Divisible  
 Dividend

group      groups of  
          lots of      divide  
 divided by      quotient  
 division      factor  
          remainder      divisible  
 half      halve      share